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A New Methodology for the Acoustic Design of Compression Driver Phase-Plugs with Concentric Annular Channels

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ABSTRACT

In compression drivers a large membrane is coupled to a small horn throat resulting in high efficiency. For this efficiency to be maintained to high frequencies the volume of the resulting cavity, between horn and membrane, must be kept small. Early workers devised a phase-plug to fill most of the cavity volume and connect membrane to horn throat with concentric annular channels of equal length to avoid destructive interference [1].

Later work, representing the cavity as a flat disc, describes a method of calculating the positions and areas of these annular channels where they exit the cavity, giving least modal excitation, thus avoiding undesirable response irregularities [2].

In this paper the result of applying both the equal path-length and modal approaches to a phase-plug with concentric annular channels coupled to a cavity shaped as a flat disc is further explored. The assumption that the cavity may be represented as a flat disc is investigated by comparing its behavior with that of an axially vibrating rigid spherical cap radiating into a curved cavity.

It is demonstrated that channel arrangements derived for a flat disc are not optimum for use in a typical compression driver with a curved cavity.

A new methodology for calculating the channel positions and areas giving least modal excitation is described. The impact of the new approach will be illustrated with a practical design.

1. INTRODUCTION

Compression drivers couple a radiating membrane to a horn throat of smaller area. When coupled to a suitable horn, this area reduction improves the match between the mechanical impedance of the driver and the acoustic radiation impedance. This improved impedance matching results in efficiencies far higher than those produced by direct radiators.

Early workers [3] found that achieving this high compression ratio required the cavity between horn throat and membrane to be 'plugged' reducing the air volume and high frequency loss due to its compliance. Further improvement at high-frequencies could be made by splitting the sound path between membrane and throat into a number of channels of equal length and of smoothly increasing area. These devices soon became known as 'phase-plugs' since they were intended to correct the phase by providing paths of equal length from the membrane to the horn throat thus avoiding destructive interference. Phase-plugs allow large membranes and correspondingly large voice coils to be used with directional horns yielding a massive increase in maximum SPL when compared to an equivalent direct radiating loudspeaker.

Poorly designed phase-plugs exhibit large response irregularities which often limit driver performance.

In this paper we firstly explore 'equal path-length' and 'modal' methods of phase-plug design on an idealized model with a cylindrical air cavity and planar rigid piston using the Finite Element Method (FEM). We will then compare this flat disc shaped cavity to a more realistic spherical cap shaped cavity. A new methodology for the positioning and sizing of annular phase-plug channels is then presented.

1.1. Compression Driver Description

Figure 1 shows a schematic section through a typical compression driver with annular channel phase-plug. The ring magnet produces an intense magnetic field that is concentrated in the magnetic gap by the highly permeable top plate and yoke. The voice coil is immersed in this strong radial field, current flowing in it results in an axial force. This force is transmitted to the membrane by a rigid former. A flexible suspension maintains the centralization of the coil, while allowing axial motion. Axial displacement of the membrane causes the cavity width to be reduced, resulting in an airflow down the channels, through the driver exit and into the horn. In the driver considered in this paper, the

magnetic gap is filled with magnetic fluid. This simplifies the acoustic behavior by blocking the air cavity in the magnet.

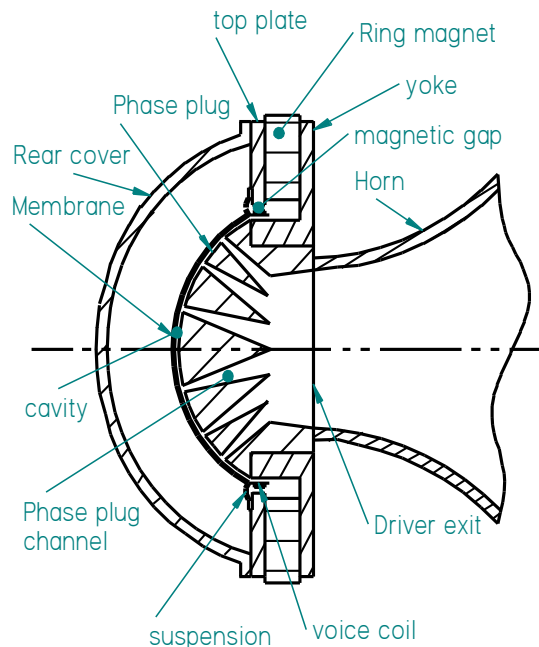


Figure 1: Diagram of compression driver

1.2. FEM Methodology

The geometries being considered in this paper are all axisymmetric and orientated with the x-axis being the axis of rotational symmetry. To take advantage of this symmetry in the analysis, second-order axisymmetric elements are used throughout.

The models of idealized compression-driver membranes are represented with shell elements, having repeated freedoms and restraints to replicate rigid body motion. The air is represented by 'solid' quadrilateral elements. The termination of the channels is achieved by applying the specific impedance of air as a boundary condition to the channel ends. This avoids reflections as the channels are narrow compared to a wavelength and only plane waves can propagate within them. The effects of viscosity are neglected in these models.

The models of idealized compression drivers have a constant force applied to the membrane, whereas the models of the cavity only have the membrane constrained to move a fixed displacement.

The FEM examples of idealized drivers have a moving mass of 0.04 gram, with 1N of driving force is applied.

All models have a 40mm diameter membrane with cavity width of 0.3mm and a compression ratio, which is the ratio of total channel area to membrane area, of 15.

The solutions are obtained with the PAFEC [4] steady state solver running on an Intel Xeon 2.33Ghz processor. For the idealized compression drivers this allowed 500 frequencies to be solved in 20 seconds.

The idealized driver models are solved to obtain the frequency responses of the pressure at selected points near the channel end. For our purposes, the aim is to have smooth responses with the same pressure in all channels.

The models of cavity without channels have the frequency response extracted over a range of positions expressed as radii for the flat disc or cavity angle in the curved cases. Contour plots of the pressure in the cavity as a function of frequency and position are produced to illustrate the pressure behavior in the cavity.

2. CURRENT PHASE-PLUG DESIGN METHODS

2.1. Equal Path Length Approach

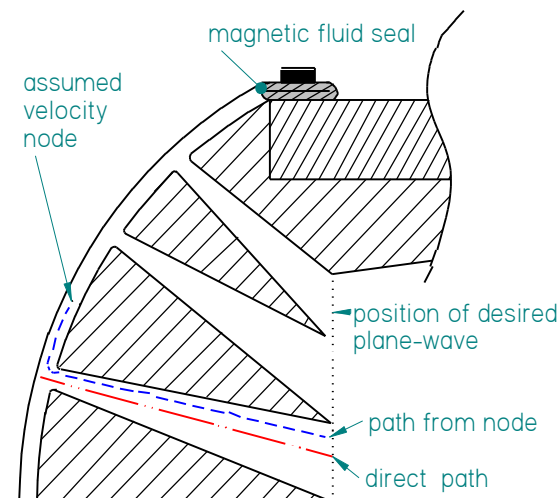


Figure 2: Detail of compression driver showing path lengths from membrane to position of desired wavefront.

One of the earliest concepts applied to positioning the channels in a phase-plug results from considering the effect of destructive interference. If we assume that the sound simply travels from a point on the membrane to the nearest channel then we can consider the path lengths from the membrane to the desired wavefront. It

would then seem reasonable to assume that where the shortest and longest path lengths are half a wavelength we could expect destructive interference to occur. This leads to the conclusion that if we position the channels to keep the path length differences to less than a quarter of a wavelength of the highest frequency in the desired bandwidth then we could expect a good response up to this frequency [5].

To explore this idea further we will consider the results of a FEM model of an idealized version of the compression driver shown in Figure 1. Compression drivers with magnetic fluid in the gap fit the idealized model best since the fluid may be assumed to remain stationary. The resulting acoustical seal, shown in Figure 2, prevents sound leakage out of the cavity between membrane and phase-plug.

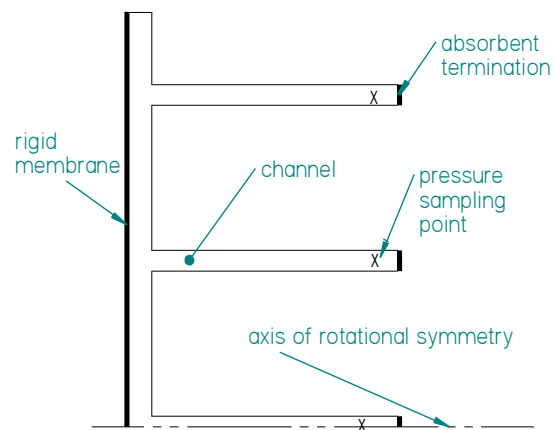


Figure 3: Geometry used for FEM model with three evenly spaced channels

The idealized driver, shown in Figure 3, has three channels and circular planar piston. The piston is constrained to move as a rigid body and forms one face of the cylindrical cavity. The other face has the three equally spaced annular channels terminated with an appropriate impedance to avoid reflections.

The path length difference between longest and shortest path is 16mm, which is half a wavelength at 10.625kHz. The results of the FEM analysis are shown in Figure 4, the pressure in the three channels is plotted.

It may be observed that these pressures are not regular and do not all have dips at 10kHz as one may expect. Additionally, some severe peaks are present in the response that cannot be readily explained by path length difference and interference.

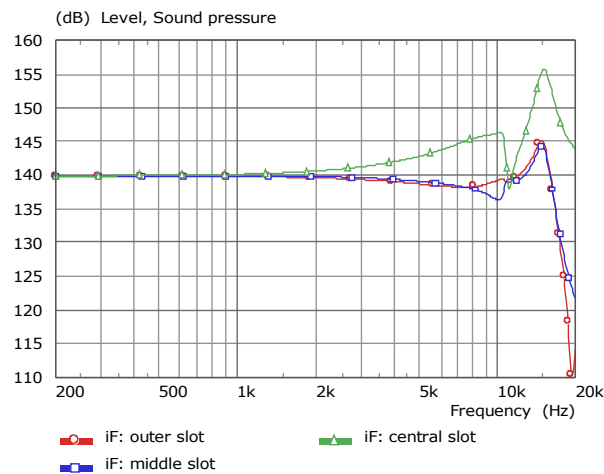


Figure 4: Pressure response of cylindrical cavity with equal channel widths and equal path length.

2.2. Modal Control by Channel Balancing

When a phase-plug is placed in front of the radiating membrane, a thin cavity is created. The response irregularities observed in section 2.1 are due to excitation of acoustic resonances in this cavity.

The nature of these resonances can be clearly demonstrated if the same cavity as analyzed above, with channels removed, is driven by a source at its outside diameter. Figure 5 shows a contour plot the pressure variation across the cavity radius (vertical axis) with frequency (horizontal axis) calculated by an FEM model for this case.

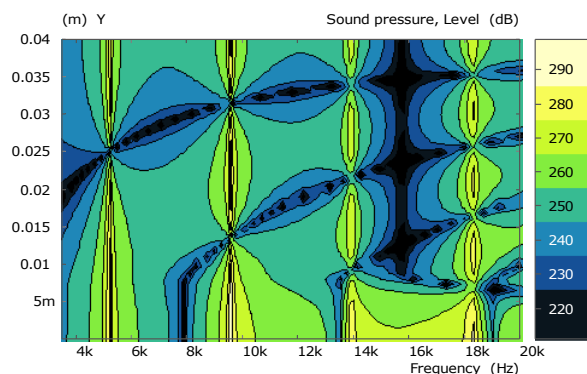


Figure 5: Pressure variation in cavity with source at outside diameter.

2.2.1. Modal Excitation by Channels Alone

An interesting solution exists when the same cylindrical cavity with no channels is driven by a circular planar

rigid membrane on one of its faces. The piston is constrained to move with a fixed displacement to avoid pressure variation with frequency. The pressure variation across the cavity radius is displayed as a contour in Figure 6. There is a maximum of 0.002dB variation across the entire cavity.

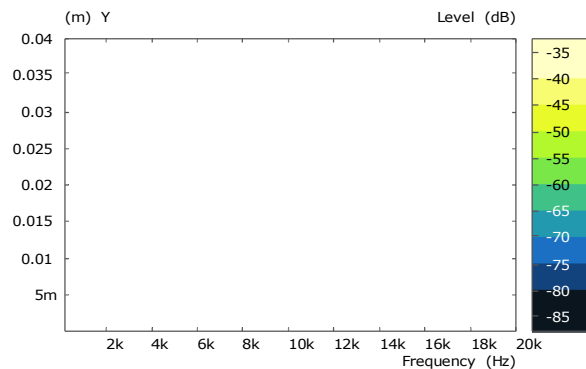


Figure 6: Planar piston in cylindrical cavity. No contours since pressure variation only 0.002dB.

There is no modal excitation since axially the cavity is much smaller than a wavelength, even at the highest frequency considered, and the radial modes are not excited as they are driven orthogonally.

From this result we may now conclude that, in the case of the evenly spaced channels, discussed in section 2.1, it is the loading of the channels that excites the cavity modes.

2.2.2. Suppression of Modes by Channel Position and Size

Smith [2] shows that it is possible, in this simplified cylindrical representation, to choose the areas and radii of the N channels so that the modal excitation from each cancels for modes up to the N^{th} mode. An example of such a design is illustrated by Murray [6].

Figure 7 shows the excitation of the first radial cavity mode by each of three phase-plug channels positioned as described by Smith. The three modal excitations, due to the presence of each channel, sum to zero and the mode is suppressed.

Figure 8 displays the same information, this time for the second radial cavity mode. The degree to which each channel excites the mode is dependent upon the channel area and also the magnitude of the mode at the channel position, this is why the magnitude of excitation from each channel is not the same in the two plotted cases.

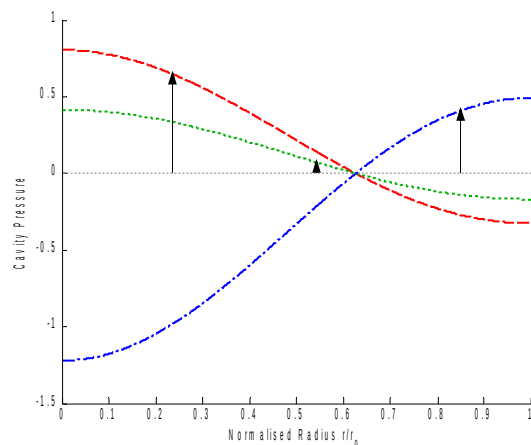


Figure 7: Pressure excited by each channel plotted against normalized radius for the first mode.

The third radial mode is also suppressed, though, this is not plotted as it is a trivial result - each channel is positioned at a nodal line and each provides no contribution to the modal excitation.

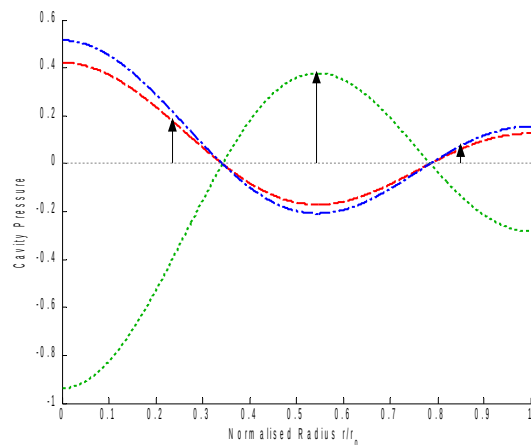


Figure 8: Pressure excited by each channel plotted against normalized radius for the second mode

The vectors show the excitation of the modes at the channel positions. In all three cases, at any position in the cavity, the pressure cancels resulting in zero modal excitation.

The interested reader is directed to section 6.1 in the appendix.

Figure 9 shows the pressure response of each channel calculated by an FEM model of a three channel Smith phase-plug. The channel pressures are almost identical confirming the almost complete absence of cavity modes.

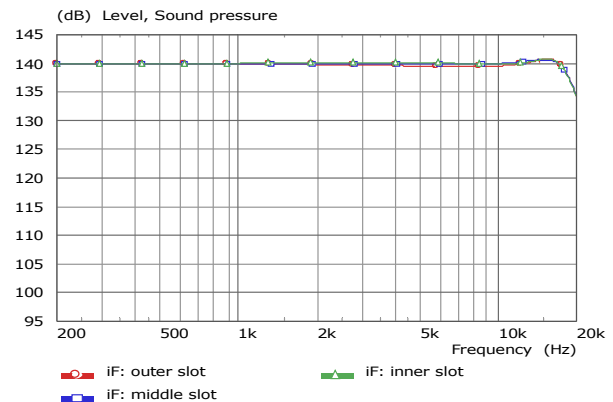


Figure 9: Pressure response of planar piston with phase-plug design to suppress modes.

2.2.3. The Effect of Practical Departure from a Cylindrical Geometry

In practice compression driver membranes need curvature to be sufficiently rigid and are shaped as a spherical cap. The more curved the membrane the higher the first parasitic structural mode.

The effect of this departure from the simplified cylindrical geometry cannot be neglected: the motion of the membrane is no longer orthogonal to the cavity resonances.

Figure 10 shows that an axially moving spherical cap will excite resonances in the, now curved, compression cavity. The positions and ratios calculated by Smiths analysis will not suppress this modal excitation.

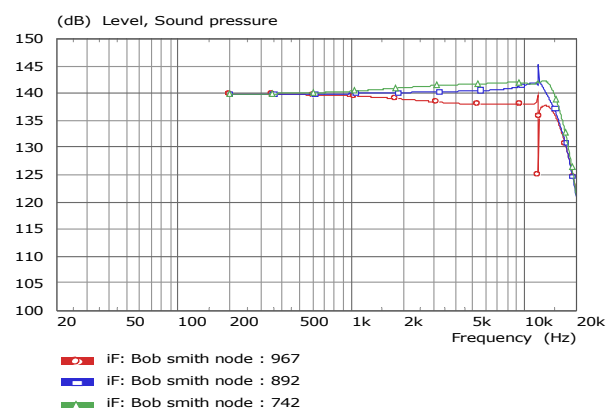


Figure 10: Spherical cap membrane with channels defined by applying result for cylindrical cavity

2.3. Modal Excitation by Membrane

The important aspect of the behavior that is overlooked by an analysis in cylindrical coordinates is the excitation of the cavity modes directly from the membrane motion. Figure 11 shows the pressure in a curved cavity with no channels excited by an axially moving spherical cap.

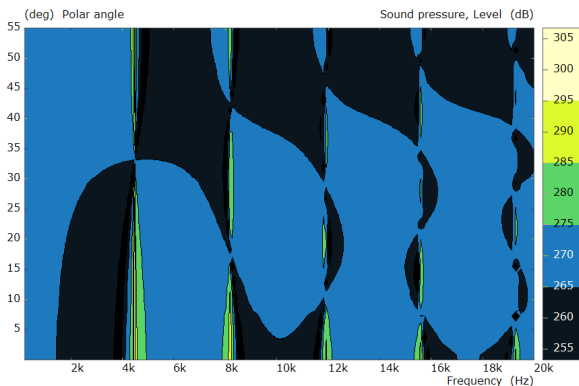


Figure 11: FEM model of curved cavity, no channels, rigid axially moving membrane.

Unlike the planar piston the resulting volume velocity is not orthogonal to the modes and the cavity resonances are excited.

2.4. Suppression of Modes by Balancing Channel and Membrane Excitation

In order to improve upon Smiths method for setting channel geometry, it is necessary to analyze the behavior of the membrane, cavity and channels in a spherical coordinate system. This allows a closer representation to the practical reality since the cavity and membrane are spherical caps just as in real compression drivers.

Conceptually, the approach is similar to Smiths. All excitation of the cavity modes is synthesized such that the overall pressure variation in the cavity is zero. Whereas in Figures 7 and 8 all contributions are due to acoustic velocity at the channel entrances, we now have an additional contribution due to the velocity of the membrane itself.

Figures 12, 13 and 14 show the modal excitations from channels and membrane for a 55 degree design. The channel positions and areas are designed so that the modal excitation due to each channel and also due to the membrane sum to give zero pressure variation in the cavity.

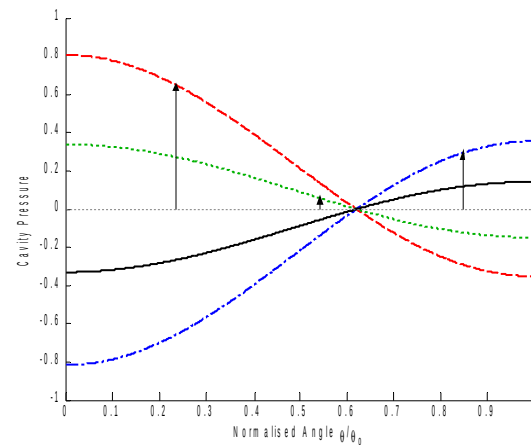


Figure 12: Pressure excited by each channel (broken lines) and membrane (solid line) plotted against normalized cavity angle for the first mode.

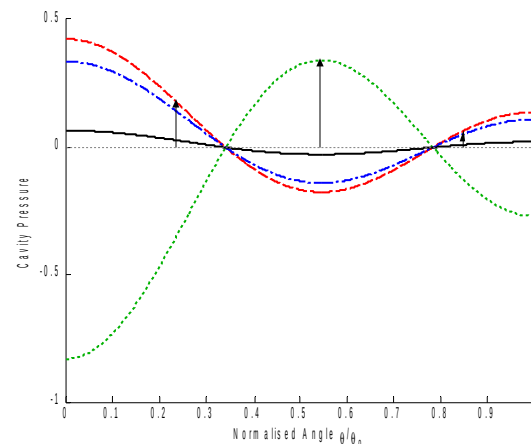


Figure 13: Pressure excited by each channel (broken lines) and membrane (solid line) plotted against normalized cavity angle for the second mode.

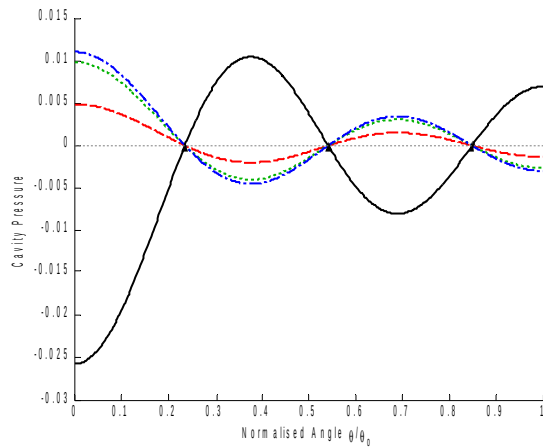


Figure 14: Pressure excited by each channel (broken lines) and membrane (solid line) plotted against normalized cavity angle for the third mode.

Derivation of the positions and areas of channel required to achieve this modal suppression was carried out by two different methods.

Firstly, the curved cavity was analyzed using a parameterized FEM model. Among the parameters of the model were the design variables in which we are interested: the position and area of the channels. An optimization was used to minimize the difference between the three channel pressures.

Secondly, the behavior of the cavity was analyzed analytically in a similar manner to Smith albeit in spherical coordinates. The details of this analysis are given in Appendix I, section 6.2.

In both cases very similar results were obtained. Unlike the Smith design guidelines, the channel geometry is dependent upon the angle of the membrane and cavity. The degree to which the membrane itself excites the cavity modes is dependent upon the cavity angle. The new design method tends towards the Smith guidelines as the angle of the cavity curvature is decreased and the flat, cylindrical situation is approached.

55 degree	Flat disc
$0.235 \theta_0$	$0.238 r_0$
$0.543 \theta_0$	$0.543 r_0$
$0.848 \theta_0$	$0.853 r_0$

Table 1: Comparison of channel positions for a three channel design having a 55 degree curved cavity and a flat cavity [7].

55 degree	Flat disc
1	1
0.9056	1.025
0.6973	1.065

Table 2: Comparison of channel width for a three channel design having a 55 degree curved cavity and a flat cavity [7].

It may be observed from tables 1 and 2 that the change in the calculated widths of the channels is much larger than the change in the positions. In both cases, we position our channels to be very close to the nodal lines of the N^{th} cavity mode: the similarity in positioning highlights that in both cases the mode shape is very similar and these nodal lines occur in roughly the same position around the cavity.

Being a little more precise, with the new design method the target is, in fact, not to place the channels in *exactly* the nodal positions but, as with the lower modes, to compensate for the excitation by the membrane. The channels are placed slightly away from the nodal lines. However, this shift is very small as inevitably the channels excite the N^{th} mode constructively, additionally the membrane modal excitation reduces with higher modes: only a minute shift is required to match the channel excitation to the membrane excitation. In practice, it is not clear if the best solution is to attempt to suppress the N^{th} mode or to simply decouple the channels from it and aim positionally directly for the nodal lines. This is largely academic with any reasonable dome angle and the cases $N > 1$. In practical terms the positional difference is minute, as can be seen from Table 1. For the case $N=1$, however, this question remains.

Figure 15 shows the pressures in the three channels calculated by an FEM model of the same curved cavity as in Figure 10 with channels arranged using the new design method. A good improvement in the regularity of the pressure in each of the channels can be observed.

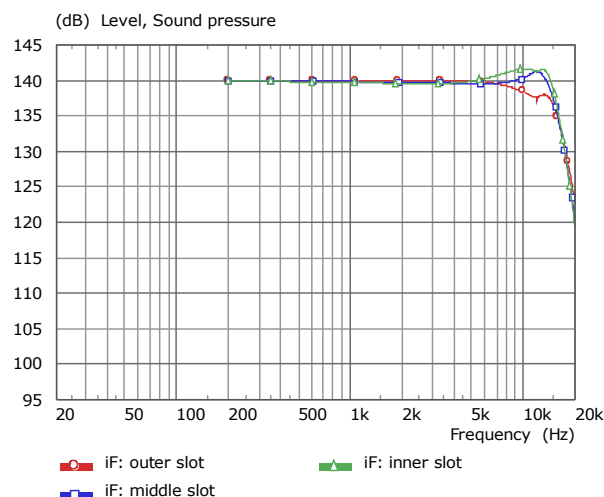


Figure 15: spherical cavity with channel position and width derived by new approach.

It is interesting to note that it is not possible to synthesize a design to work as well in the curved cavity as can be achieved in the flat disc case, Figure 9. With the flat disc cavity, it is not necessary to link the channel velocity to the membrane velocity as all excitation of modes is by the channels. With the curved case, this added complication results in a small quadrature component between the channel and membrane velocity that cannot be suppressed, see 6.2.4. Additionally, it is very safe to assume that, in the case of the flat disc, the acoustical impedance of each of the channels is the same; however, this is certainly a less accurate assumption in the curved case as the channels themselves are forced to have some curvature. Nevertheless, it is clear that the assumptions made in the analytical analysis are quite reasonable because of the good agreement with the optimized FEM results.

3. APPLICATION TO PRACTICAL COMPRESSION DRIVER.

The approach described in this paper was developed during the design of a 1.4 Inch diameter exit compression driver. This driver has a 3 inch voice coil wound with copper-clad aluminium wire on to a glass filled polyimide former. The former is coupled to a titanium membrane, which is supported by a polyimide surround.

The acoustic FEM model used in the driver development includes most physical detail. The mesh used was associatively linked to a parameterized solid model. This allowed most aspects of the design to be

readily varied. For example channel position, width and length were all controlled by parameters in addition to many other aspects of the design.

Mechanical parts are all flexible and are coupled to the acoustical parts of the model.

Many FEM models of other domains were used, static magnetic for magnet design. Magneto-dynamic for copper sleeve design and derivation of blocked impedance of voice coil. The use of these other FEM techniques to derive a frequency response of a driver are described in [8]. Additionally, large displacement analysis was used to evaluate the compliance variation with diaphragm displacement due to the surround resulting in the choice of a small surround with a single roll.

One important aspect of the phase-plug design was the length and area of the channels themselves. While the application of path lengths to the positions of the channels was not found to give a useful result, the length of the channels themselves was found to be important. This is illustrated as an example in [8].

The final phase-plug design is illustrated in Figure 16 mounted on a horn.

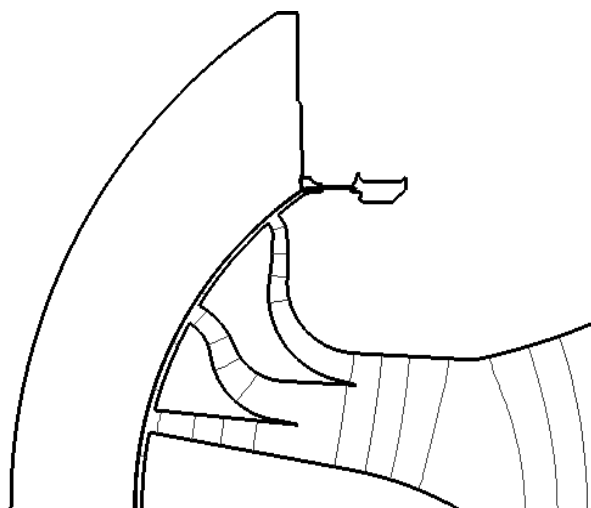


Figure 16: FEM model of final design on horn showing iso-pressure contours at 8,417Hz.

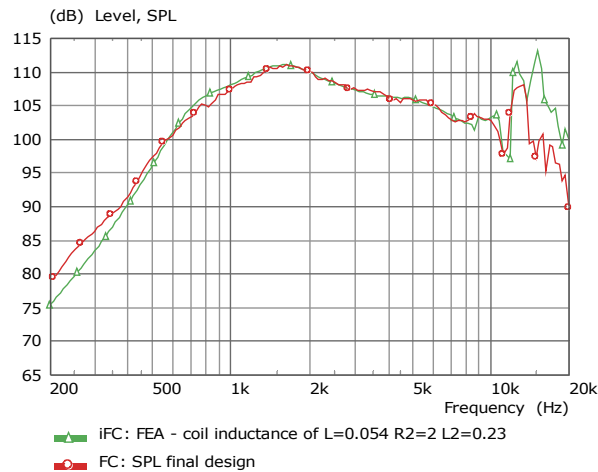


Figure 17: FEM versus measured frequency response on axisymmetric horn.

4. DISCUSSION

This work, by necessity rather than intention, has resulted in an interesting counterpoint of analytical and computational analysis.

FEM used as a tool to get at the behavior by splitting the problem/design into abstract experiments.

Analytical analysis helps to confirm the physics and that the optimum solution is reached. It also makes it possible to generalize (for example for many angles). For some cases with too many degrees of freedom there is no alternative.

FEM can, of course, go further and attempt to find solutions for more complex cases such as optimum positions and widths of channels when the membrane is not rigid or cavity and membrane shapes that cannot be described by separable coordinate systems.

Computational analysis is not, however, required to understand that the cylindrical representation is not a sufficient description of the problem considered. Though, conversely, the analytical analysis of the spherical case is complicated by the presence of Legendre functions of non-integer order and integrals of these functions. Issues dealing with such expressions are neatly sidestepped as happily we are able to insert a little numerical analysis when other routes fail. The Eigenfunctions used in the analysis are, in fact, calculated with FEM models.

The traditional analytical analysis approach of reducing a problem into easy-to-understand simply behaving parts is a truly powerful way of applying FEM modeling. The parts can be analyzed and conquered individually. Assumptions can be checked and the parts re-assembled after they are understood.

5. CONCLUSIONS

The equal path-length approach does not take into account the modal behavior and cannot be expected to produce optimum results in every case. Smith's methodology is very successful with the flat disc shaped cavity giving almost total suppression of the modes. However, when applied to a cavity shaped like a spherical cap Smith's technique does not produce such good results; While for the planar membrane, the channels alone excite the modes; the spherical cap membrane excites the modes even in the absence of channels.

In this paper it is demonstrated that the correct choice of channel widths and positions can compensate for the normal volume velocity variation of the membrane and avoid exciting modes.

6. APPENDIX I – ANALYTICAL ANALYSIS OF CYLINDRICAL AND CURVED CAVITY

An analytical analysis of the compression cavity is presented in this section. In addition to the analysis in a spherical co-ordinate system, as described in the main body of the paper, Smith's analysis in a cylindrical cavity is repeated in the same terms as the new analysis so that the two methods may be directly compared.

The driven behavior an acoustical enclosure can be described as summation in the form shown below in equation 1.

$$p(\bar{\mathbf{x}}) = \sum_{n=0}^{\infty} \frac{\omega \rho_o c_o^2 \Psi_n(\bar{\mathbf{x}})}{V [\omega c_o D_{nn} + j(\omega^2 - \omega_n^2)]} \int_s \Psi_n(\bar{\mathbf{y}}) u(\bar{\mathbf{y}}) \cdot \bar{\mathbf{n}} dS \quad (1)$$

This form of expression, sometimes referred to as a modal decomposition, was described by Morse and Bolt. A clear explanation and modern derivation of the expression can be found by Nelson and Elliot [9].

The terms $\Psi_n(\bar{\mathbf{x}})$ and ω_n above are the Eigenfunctions and Eigenvalues of the enclosure respectively.

6.1. Analysis with Cylindrical Geometry

The compression cavity is analyzed in a simplified cylindrical geometry as illustrated in Figure 18. This is a repetition of the analysis performed by Smith [2].

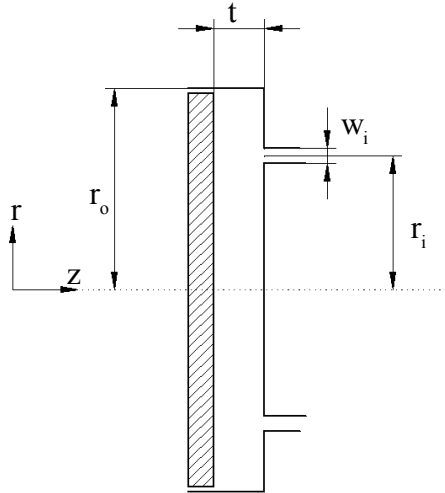


Figure 18: Cylindrical representation of compression driver.

6.1.1. Eigenfunction and Eigenfrequency Calculation

Starting from the Helmholtz equation

$$\nabla^2 p + \frac{\omega^2}{c_o^2} p = 0 \quad (2)$$

In cylindrical coordinates the Laplacian can be written as shown below in equation 3 [10].

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \quad (3)$$

The compression chamber is a thin disc. In the operating range of the driver the variation of pressure across the thickness of the disc is negligible. Additionally, the cavity, membrane and channel entrances are rotationally symmetric. We can remove some terms from the Laplacian to give equation 4.

$$\nabla^2 p = \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \quad (4)$$

Substituting equation 4 into equation 2 and multiplying by r^2 .

$$r^2 \frac{\partial^2 p}{\partial r^2} + r \frac{\partial p}{\partial r} + r^2 \frac{\omega^2}{c_o^2} p = 0 \quad (5)$$

Bowman [11] describes a transformed version of the Bessel differential equation [12], a simplified version of which is shown in equation 6. The similarity of 5 with 6 is clear.

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + x^2 \alpha^2 y = 0 \quad (6)$$

Equation 6 has solutions

$$y = C_1 J_0(\alpha x) + C_2 Y_0(\alpha x) \quad (7)$$

Where J_0 is a Bessel function of the first kind and Y_0 is a Bessel function of the second kind. Y_0 is singular at $Y_0(0)$. Constraining the pressure in the cavity to be finite, we shall assume that the pressure Eigenfunction may be described as shown in equation 8.

$$\Psi_n(r) = C_1 J_0(\alpha r) \quad (8)$$

The pressure must obey the rigid wall boundary condition at the diameter of the cylindrical cavity as described in equation 9.

$$\left[\frac{d\Psi_n(r)}{dr} = 0 \right]_{r=r_o} \quad (9)$$

This requirement can be met by choice of α as shown in equation 10 where j_n is the n^{th} root of the Bessel function of the first kind J_1 .

$$\alpha_n = \frac{j_n}{r_o}, \quad J_1(j_n) = 0 \quad (10)$$

The eigenfrequencies can be found by equating $\alpha^2 = \omega^2 / c_o^2$.

6.1.2. Pressure Response of Cavity without Channels

Having calculated the rigid walled Eigenfunctions and frequencies above we may now analyze the driven behavior of the compression cavity. It is interesting to first perform this analysis for a cavity having a rigid-disc membrane radiator occupying the entire entrance side and no channels on the exit side.

The pressure in the cavity can, as discussed in section 6, be described as a summation of contributions from each of the calculated Eigenfunctions. We neglect the Dnn damping term present in equation 1 as, in the rigidly walled compression cavity, this is vanishingly small.

$$p(\bar{\mathbf{x}}) = \frac{j\omega\rho_o c_o^2}{V} \sum_{n=0}^{\infty} \frac{\Psi_n(\bar{\mathbf{x}})}{\omega_n^2 - \omega^2} \int \Psi_n(\bar{\mathbf{y}}) u(\bar{\mathbf{y}}) \cdot \bar{\mathbf{n}} dS \quad (11)$$

The cavity is only driven by the membrane on the entrance side of the compression cavity, the surface velocity is zero at all other locations on the enclosure walls. The integral on the right hand side can be re-written by change of variables, u_o is the membrane velocity.

$$\hat{S} = \pi r^2 \rightarrow \frac{d\hat{S}}{dr} = 2\pi r$$

$$\int_S \Psi_n(\bar{\mathbf{y}}) u(\bar{\mathbf{y}}) \cdot \bar{\mathbf{n}} dS = \int_{r=0}^{r_o} \Psi_n(r) u_o 2\pi r dr \quad (12)$$

From this it is clear, because of the orthogonality of the Eigenfunctions, that this integral is only non zero for $n=0$.

$$\int_{r=0}^{r_o} \Psi_n(r) u_o 2\pi r dr = \begin{cases} u_o \pi r_o^2 & n=0 \\ 0 & n \neq 0 \end{cases} \quad (13)$$

The summation from equation 11 disappears and the pressure in the cavity is simply

$$p = \frac{\rho_o c_o^2}{j\omega V} u_o \pi r_o^2 \quad (14)$$

The pressure field in the cavity does not have contributions from any mode other than the zeroth, the static 0Hz mode. This is because the excitation is orthogonal to all other calculated Eigenfunctions.

6.1.3. Pressure Response of Cavity with a number of Annular Exit Channels

The arrangement of a compression driver is to have not only a radiating membrane on one face of the cylindrical compression cavity, but also a number of exit channels on the opposite face, which lead through the phase-plug and to the horn, through which sound is radiated.

The exit side surface normal acoustic velocity is represented by the function $f(r)$ defined as below in equation 15.

$$f(r) = \begin{cases} 0 & r \neq r_1, r \neq r_2, r \neq r_3 \dots \\ u_1 & r = r_1 \\ u_2 & r = r_2 \\ \dots & \end{cases} \quad (15)$$

We may extend equation 14 to include pressure variations in the cavity occurring due to the velocity $f(r)$.

$$p(r) = \frac{j\omega\rho_o c_o^2}{V} \left[\frac{u_o \pi r_o^2}{-\omega^2} + \sum_{n=0}^{\infty} \frac{\Psi_n(r)}{\omega_n^2 - \omega^2} \int_S \Psi_n(\hat{r}) f(\hat{r}) dS \right] \quad (16)$$

As the channels are narrow, we approximate that the integral may be evaluated as shown in equation 17 with reasonable accuracy.

$$\int_S \Psi_n(\hat{r}) f(\hat{r}) dS = \sum_{i=1}^N \Psi_n(r_i) A_i u_i \quad (17)$$

A_i in equation 17 is the area of the i^{th} channel entrance, r_i is the radial location of the channel and u_i is the acoustic velocity at the channel entrance.

$$p(r) = \frac{\rho_o c_o^2}{j\omega V} u_o \pi r_o^2 + \frac{j\omega\rho_o c_o^2}{V} \sum_{n=0}^{\infty} \frac{\Psi_n(r)}{\omega_n^2 - \omega^2} \sum_{i=1}^N \Psi_n(r_i) A_i u_i \quad (18)$$

6.1.4. Suppression of Modal excitation by Channel Arrangement

By separating the zeroth term from the first summation, as in equation 19, we can shown the pressure in the cavity in terms of the lumped desired behavior plus undesirable terms due to excitation of the cavity modes due to the velocity in the compression channels.

$$p(r) = \frac{\rho_o c_o^2}{j\omega V} \left[u_o \pi r_o^2 + \sum_{i=1}^N A_i u_i \right] + \frac{j\omega\rho_o c_o^2}{V} \sum_{n=1}^{\infty} \frac{\Psi_n(r)}{\omega_n^2 - \omega^2} \sum_{i=1}^N \Psi_n(r_i) A_i u_i \quad (19)$$

From this expression it is clear that in order to suppress excitation of a mode, m , we require that

$$\sum_{i=1}^N \Psi_m(r_i) A_i u_i = 0 \quad (20)$$

It is assumed that we wish to suppress the first N modes, where N is the number of annular channels in the cavity, and thus enforce the lumped behavior to as high a frequency as possible.

In this case we assume that if our geometry provides good modal suppression the membrane velocity and the cavity pressure can be related simply by the lumped terms as in equation 21.

$$p = \frac{\rho_o c_o^2}{j\omega V} \left[u_o \pi r_o^2 + \sum_{i=1}^N A_i u_i \right] \quad (21)$$

The velocities at the channel entrances can be related to the cavity pressure at the channel entrance by the acoustic impedance of the channel.

$$u_i = \frac{p}{z_i} \quad (22)$$

In the mode suppressed case the pressure in the cavity is constant. If we assume that the acoustic impedance of each channel is identical it must follow that the acoustic velocity in each channel entrance is identical. Equation 20 simplifies as shown in equation 23.

$$\sum_{i=1}^N \Psi_m(r_i) A_i = 0 \quad (23)$$

This condition can be met by choice of the channel entrance areas, A_i , and the channel positions, r_i . The easiest way to achieve this is to set r_i to the nodal positions of the highest mode that we wish to suppress. This then leaves $N-1$ equations with $N-1$ unknowns, which can be solved to find the ratios of the areas of the channels.

For example in the case of a three-channel phase-plug we choose:

$$\begin{aligned} r_1 &= 0.238 r_o \\ r_2 &= 0.543 r_o \\ r_3 &= 0.853 r_o \end{aligned} \quad (24)$$

Solving the simultaneous equations as described above gives the corresponding area ratios:

$$\begin{aligned} \frac{A_3}{A_1} &= 3.817 \\ \frac{A_2}{A_1} &= 2.3386 \end{aligned} \quad (25)$$

Which can be equivalently written as channel width ratios:

$$\begin{aligned} \frac{w_3}{w_1} &= 1.065 \\ \frac{w_2}{w_1} &= 1.025 \end{aligned} \quad (26)$$

This result is a repetition of that demonstrated by Smith [2].

6.2. Analysis with Spherical Geometry

The cavity analysis is repeated in a spherical coordinate system, approximating the geometry as shown in Figure 19.

6.2.1. Eigenfunction and Eigenfrequency Calculation

Starting from the Helmholtz equation

$$\nabla^2 p + \frac{\omega^2}{c_o^2} p = 0 \quad (27)$$

In spherical coordinates, the Laplacian can be written as shown below in equation 28 [13].

$$\nabla^2 p = \frac{\partial^2 p}{\partial h^2} + \frac{2}{h} \frac{\partial p}{\partial h} + \frac{1}{h^2 \sin^2 \phi} \frac{\partial^2 p}{\partial \theta^2} + \frac{1}{h^2} \frac{\partial^2 p}{\partial \phi^2} + \frac{\cos \phi}{h^2 \sin \phi} \frac{\partial p}{\partial \phi} \quad (28)$$

The compression chamber is thin in h . In the operating range of the driver, the variation of pressure across the thickness of the cavity is negligible. Additionally, the cavity, membrane and channel entrances are rotationally symmetric. We can remove some terms from the Laplacian to give equation 29.

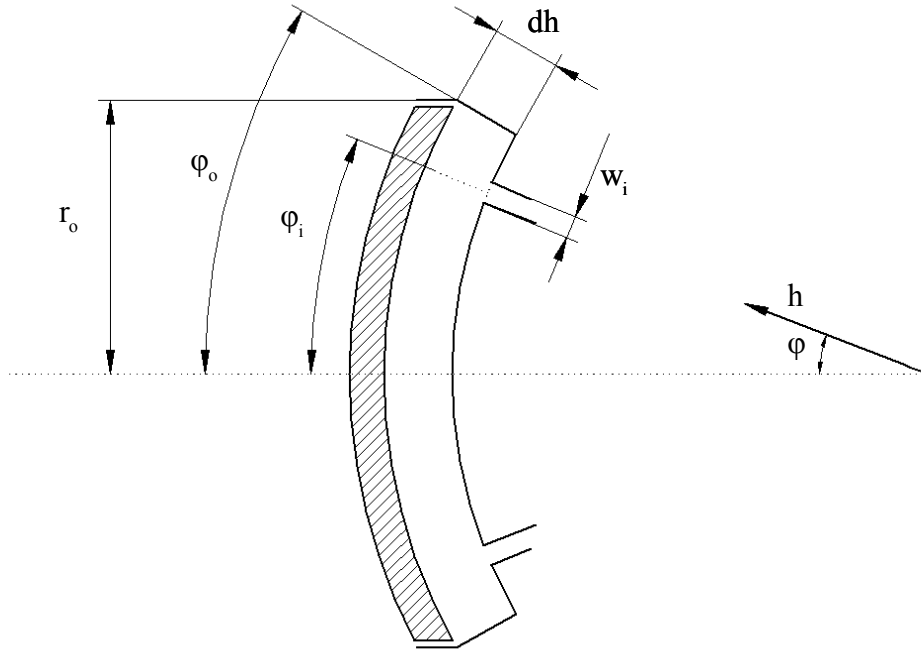


Figure 19: Spherical representation of compression driver.

$$\nabla^2 p = \frac{1}{h^2} \frac{\partial^2 p}{\partial \phi^2} + \frac{\cos \phi}{h^2 \sin \phi} \frac{\partial p}{\partial \phi} \quad (29)$$

Substituting equation 29 into equation 27 and multiplying by h^2 .

$$\frac{\partial^2 p}{\partial \phi^2} + \frac{\cos \phi}{\sin \phi} \frac{\partial p}{\partial \phi} + \frac{h^2 \omega^2}{c_o^2} p = 0 \quad (30)$$

A form of the associated Legendre differential equation written for $x = \cos \phi$ is written in equation 31 [14].

$$\frac{\partial^2 y}{\partial \phi^2} + \frac{\cos \phi}{\sin \phi} \frac{\partial y}{\partial \phi} + \iota(\iota+1)y = 0 \quad (31)$$

The similarity of 30 with 31 is clear. Equation 31 has two linearly independent solutions as shown in equation 32.

$$y = C_1 P_\iota(\cos \phi) + C_2 Q_\iota(\cos \phi) \quad (32)$$

$P_\iota(x)$ is a Legendre function of order ι which is finite for all values of x . $Q_\iota(x)$ is a Legendre function of order ι which is singular for $x = \pm 1$. Note that, as we cannot guarantee that ι is an integer, we cannot assume that these functions are Legendre polynomials.

Constraining the pressure in the cavity to be finite, we shall assume that the pressure Eigenfunction may be described as shown in equation 33.

$$\Psi_n(\phi) = C_1 P_\iota(\cos \phi) \quad (33)$$

The pressure must obey the rigid wall boundary condition at the diameter of the cavity as described in equation 34.

$$\left[\frac{d \Psi_n(\phi)}{d \phi} \right]_{\phi=\phi_0} = 0 \quad (34)$$

This requirement can be met by choice of $\mathbf{l}$. Hoersch demonstrates that a Legendre function of the first kind may be approximately described as a summation of Bessel functions [15]. Using this expression for the Legendre function in equation 33, it is possible to analytically derive values of $\mathbf{l}_n$ that solve 34 in terms of a summation of Bessel function roots. The eigenfrequencies can then be found as shown in equation 35.

$$\mathbf{l}_n(\mathbf{l}_n+1) = \frac{h^2 \omega_n^2}{c_o^2} = r_o^2 \frac{\omega_n^2}{c_o^2 \sin^2 \phi_o} \quad (35)$$

Hoersch's Bessel summation description of Legendre functions is approximate and only valid for a small range of x and $\mathbf{l}$. To improve the accuracy of the computation, FEM derived values of ω_n and $\Psi_n(\phi)$ are actually used to generate the final channel position and widths in section 6.2.4.

6.2.2. Pressure Response of Cavity without Channels

As before, in section 6.1.2, having calculated the rigid walled Eigenfunctions and frequencies we may now analyze the driven behavior of the compression cavity. This is first considered for the situation where there are no exit channels present.

The pressure in the cavity can, as discussed in section 6, be described as a summation of contributions from each of the calculated Eigenfunctions. We neglect the Dnn damping term present in equation 1 as, in the rigidly walled compression cavity, this is vanishingly small.

$$p(\bar{\mathbf{x}}) = \frac{j \omega \rho_o c_o^2}{V} \sum_{n=0}^{\infty} \frac{\Psi_n(\bar{\mathbf{x}})}{\omega_n^2 - \omega^2} \int \Psi_n(\bar{\mathbf{y}}) u(\bar{\mathbf{y}}) \cdot \bar{\mathbf{n}} dS \quad (36)$$

The cavity is only driven by the membrane on the entrance side of the compression cavity, the surface velocity is zero at all other locations on the enclosure walls. The integral on the right hand side can be re-written by change of variables, u_0 is the membrane velocity.

$$\begin{aligned} \hat{S} &= \frac{2\pi r_o^2}{\sin^2 \phi_o} (1 - \cos \phi) \rightarrow \frac{d \hat{S}}{d \phi} = \frac{2\pi r_o^2}{\sin^2 \phi_o} \sin(\phi) \\ \int_S \Psi_n(\bar{\mathbf{y}}) u(\bar{\mathbf{y}}) \cdot \bar{\mathbf{n}} dS &= \frac{\pi r_o^2}{\sin^2 \phi_o} \int_{\phi=0}^{\phi_o} \Psi_n(\phi) u_o \sin(2\phi) d\phi \end{aligned} \quad (37)$$

For the case when $n=0$ this integral evaluates as shown in 38, which is the same result as for a equivalent disc membrane of the same r_o .

$$\frac{\pi r_o^2}{\sin^2 \phi_o} \int_{\phi=0}^{\phi_o} u_o \sin(2\phi) d\phi = u_o \pi r_o^2 \quad (38)$$

Substituting 37 into 36 we can write the pressure in the cavity as shown below in equation 39.

$$p(\phi) = \frac{\rho_o c_o^2}{j\omega V} u_o \pi r_o^2 + \frac{j\omega \rho_o c_o^2}{V} \sum_{n=1}^{\infty} \frac{\Psi_n(\phi)}{\omega_n^2 - \omega^2} \frac{\pi r_o^2}{\sin^2 \phi_o} \int_{\hat{\phi}=0}^{\phi_o} \Psi_n(\hat{\phi}) u_o \sin(2\hat{\phi}) d\hat{\phi} \quad (39)$$

In this case we find that in addition to the static pressure terms that we saw in the case of the cylindrical cavity, equation 14, we now also see some excitation of the cavity modes just from the diaphragm motion itself.

6.2.3. Pressure Response of Cavity with a Number of Annular Exit Channels

The arrangement of a compression driver is to have not only a radiating membrane on one face of the compression cavity, but also a number of exit channels on the opposite face.

The exit size surface normal acoustic velocity is represented by the function $f(\phi)$ defined as shown below in equation 40.

$$f(\phi) = \begin{cases} 0 & \phi \neq \phi_1, \phi \neq \phi_2, \phi \neq \phi_3 \dots \\ u_1 & \phi = \phi_1 \\ u_2 & \phi = \phi_2 \\ u_3 & \phi = \phi_3 \\ \dots & \end{cases} \quad (40)$$

We may extend equation 39 to include pressure variations in the cavity occurring due to the velocity $f(\phi)$.

$$p(\phi) = \frac{j\omega \rho_o c_o^2}{V} \sum_{n=0}^{\infty} \frac{\Psi_n(\phi)}{\omega_n^2 - \omega^2} \frac{\pi r_o^2}{\sin^2 \phi_o} \int_{\hat{\phi}=0}^{\phi_o} \Psi_n(\hat{\phi}) u_o \sin(2\hat{\phi}) d\hat{\phi} \dots \quad (41)$$

$$+ \frac{j\omega \rho_o c_o^2}{V} \sum_{n=0}^{\infty} \frac{\Psi_n(\phi)}{\omega_n^2 - \omega^2} \int_S \Psi_n(\hat{\phi}) f(\hat{\phi}) dS$$

As the channels are narrow, we approximate that the integral may be evaluated as shown in equation 42 with reasonable accuracy:

$$\int_S \Psi_n(\hat{\phi}) f(\hat{\phi}) dS = \sum_{i=1}^N \Psi_n(\phi_i) A_i u_i \quad (42)$$

Inserting this result into 41 we have.

$$p(\phi) = \frac{j\omega \rho_o c_o^2}{V} \sum_{n=0}^{\infty} \frac{\Psi_n(\phi)}{\omega_n^2 - \omega^2} \left[\frac{\pi r_o^2}{\sin^2 \phi_o} \int_{\hat{\phi}=0}^{\phi_o} \Psi_n(\hat{\phi}) u_o \sin(2\hat{\phi}) d\hat{\phi} + \sum_{i=1}^N \Psi_n(\phi_i) A_i u_i \right] \quad (43)$$

6.2.4. Suppression of Modal excitation by Channel Arrangement

By separating the $n = 0$ terms from the summations to give equation 44 we can, again, shown the pressure in the cavity in terms of the lumped desired behavior plus undesirable terms due to excitation of the cavity modes due to the velocity in the compression channels and the membrane.

$$p(\phi) = \frac{\rho_o c_o^2}{j\omega V} \left[u_o \pi r_o^2 + \sum_{i=1}^N A_i u_i \right] \dots$$

$$+ \frac{j\omega \rho_o c_o^2}{V} \sum_{n=1}^{\infty} \frac{\Psi_n(\phi)}{\omega_n^2 - \omega^2} \left[\frac{\pi r_o^2}{\sin^2 \phi_o} \int_{\hat{\phi}=0}^{\phi_o} \Psi_n(\hat{\phi}) u_o \sin(2\hat{\phi}) d\hat{\phi} + \sum_{i=1}^N \Psi_n(\phi_i) A_i u_i \right] \quad (44)$$

From this expression it is clear that in order to suppress excitation of a mode, m , we require that:

$$\sum_{i=1}^N \Psi_n(\phi_i) A_i u_i = \frac{-\pi r_o^2}{\sin^2 \phi_o} \int_{\hat{\phi}=0}^{\phi_o} \Psi_n(\hat{\phi}) u_o \sin(2\hat{\phi}) d\hat{\phi} \quad (45)$$

As before it is assumed that we wish to suppress the first N modes, where N is the number of annular channels in the cavity, and thus enforce the lumped behavior to as high a frequency as possible.

In this case we assume that if our geometry provides good modal suppression the membrane velocity and the cavity pressure can be related simply by the lumped terms as in equation 46.

$$p = \frac{\rho_o c_o^2}{j\omega V} \left[u_o \pi r_o^2 + \sum_{i=1}^N A_i u_i \right] \quad (46)$$

The velocities at the channel entrances can be related to the cavity pressure at the channel entrance by the acoustic impedance of the channel. We again assume that the acoustic impedance of each channel entrance is identical from which it follows that the entrance channel velocities must also be the same. We also make the further assumption that the channel acoustic impedance is ρc_o . Additionally, we approximate that $V \approx dh \pi r_o^2$, where dh is the cavity thickness. We can relate the membrane velocity to the channel entrance velocity by inserting these conditions into 46 to give:

$$u_o = u_i \left(j \frac{dh}{\lambda} - \frac{A_T}{\pi r_o^2} \right) \quad (47)$$

A_T is the total combined area of the channel entrances. λ is the acoustic wavelength at frequency ω . From equation 47 we can see that the real part of the channel velocities are related by the compression ratio. The quadrature component presents us with a difficulty, in order for us to balance the channel excitation of the modes to cancel with the membrane excitation of the modes the velocities associated with each must be in-phase – there is nothing that can be done to suppress this quadrature component. Looking at the expression more closely we can see that this component is insignificant when the cavity thickness compared to the acoustic wavelength is smaller than the inverse of the compression ratio. In order to continue our analysis, we assume that we can neglect this component.

The variable ζ_n is defined below in equation 48 representing the degree of the modal excitation by the membrane. This can be evaluated numerically, or by other means, for each cavity mode.

$$\zeta_n = \frac{1}{\sin^2 \phi_o} \int_{\hat{\phi}=0}^{\phi_o} \Psi_n(\hat{\phi}) \sin(2\hat{\phi}) d\hat{\phi} \quad (48)$$

Substituting the real part of equation 47 into 45 along with ζ_n and the simple relationship between the area of the individual channels and the total channel area $\sum_{i=1}^N A_i = A_T$ we can write:

$$\sum_{i=1}^N A_i (\Psi_n(\phi_i) - \zeta_n) = 0 \quad (49)$$

Modal suppression is then be achieved by meeting the condition in 49.

This can be most easily achieved by setting the channel positions ϕ_i such that:

$$\Psi_N(\phi_i) = \zeta_N \quad (50)$$

This leaves a system of N-1 equations with N-1 unknowns that can be solved to find the ratios of the areas of the channels.

For example in the case of a three channel phase-plug with $\phi_o = 55^\circ$ we can calculate:

$$\begin{aligned} \zeta_1 &= 0.0624392 \\ \zeta_2 &= -0.012609 \\ \zeta_3 &= 0.0048689 \end{aligned} \quad (51)$$

We then choose out channel positions to be:

$$\begin{aligned} \phi_1 &= 0.2350 \phi_o \\ \phi_2 &= 0.5431 \phi_o \\ \phi_3 &= 0.8476 \phi_o \end{aligned} \quad (52)$$

The simultaneous equations can then be solved to find the channel areas:

$$\begin{aligned} \frac{A_2}{A_1} &= 2.0158 \\ \frac{A_3}{A_1} &= 2.2649 \end{aligned} \quad (53)$$

Which can be equivalently written as channel width ratios.

$$\begin{aligned} \frac{w_2}{w_1} &= 0.9056 \\ \frac{w_3}{w_1} &= 0.6973 \end{aligned} \quad (54)$$

7. APPENDIX II – GLOSSARY OF SYMBOLS

j	$\sqrt{-1}$
ω	<i>angular frequency</i>
c_o	<i>acoustic wave speed</i>
ρ	<i>ambient acoustic density</i>
∇^2	<i>scalar Laplacian operator</i>
$\bar{x}, \bar{y}$	<i>coordinate vectors</i>
$\bar{n}$	<i>surface normal vector</i>
$\Psi(\bar{x})_n$	n^{th} Eigenfunction evaluated at position $\bar{x}$
ω_n	n^{th} Eigenvalue
p	<i>complex acoustic pressure</i>
V	<i>physical volume of enclosure / cavity</i>
r_o	<i>outside radius of cavity</i>
r_i	<i>radial position of i^{th} channel</i>
ϕ_o	<i>angle of cavity</i>
ϕ_i	<i>angular position of i^{th} cavity</i>
A_i	<i>area of i^{th} channel</i>
u_i	<i>acoustic velocity at channel entrance</i>
z_i	<i>acoustic impedance of channel at entrance</i>
w_i	<i>width of i^{th} channel</i>
A_T	<i>total combined area of channels</i>
N	<i>number of channels in phase plug</i>

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